CEREBRAL WHITE MATTER SEGMENTATION FROM MRI USING PROBABILISTIC GRAPH CUTS AND GEOMETRIC SHAPE PRIORS

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ABSTRACT

Study of cerebral white matter in the brain is an important medical problem which helps in better understanding of brain disorders like autism. The goal of this research is to segment the cerebral white matter from the input Magnetic Resonance Imaging (MRI) data. The present segmentation problem becomes extremely difficult due to i) the complex shape of the cerebral white matter and ii) the very low contrast between the white matter and the surrounding structures in the MRI data. We employ a novel probabilistic graph cut algorithm, where the edge capacity functions of the classical graph cut algorithm are modified according to the probabilities of pixels to belong to different segmentation classes. In order to separate the surrounding structures from the white matter, two appropriate geometric shape priors are introduced. Experimentation in 2D with 20 different datasets has yielded an average segmentation accuracy of 94.78%.

Index Terms- Probabilistic Graph Cut, Cerebral White Matter, Geometric Shape Priors, Magnetic Resonance Imaging.

1. INTRODUCTION

Segmentation of the cerebral white matter, gray matter and the deep structures such as the hippocampus, corpus callosum, brain stem, and brain ventricles have always been one of the most challenging and essential tasks in any computer assisted diagnosis system. In this work, we focus on segmenting the cerebral white matter from the input Magnetic Resonance Imaging (MRI) data, a task necessary to understand brain disorders like autism.

The most popular techniques for extracting a brain structure from the MRI data can be grouped into three broad categories, namely, a) statistical models-based segmentation, b) atlas-based segmentation, and c) deformable model-based segmentation. The main idea of statistical models-based segmentation approaches is to capture the distributions of the brain tissues (e.g., white matter, gray matter, bone and cerebrospinal fluid), and model them using a linear combination of a certain mixture of well-defined distributions. For example, see the works of Atkins and Mackiewich [1] and Farag et al. [2] The statistical approaches suffer from the fact that the tails of the distributions of the brain tissues are always overlapped. Thus, it becomes quite difficult to find thresholds that separate them accurately. Atlas-based approaches are employed for segmentation of different brain structures, based on the prior information of pre-defined brain models [3-5]. Success of these methods depends heavily on the high accuracy of registration between the atlas brain and the un-segmented brain, which may be hard to obtain in many cases. A deformable or active model is a curve in a 2D digital image or a surface in a 3D image that evolves to outline a desired object. The evolution is controlled by internal and external forces combined, together with user defined constraints, into internal and external energy terms respectively [6-7]. Zeng et al. [8] presented a coupled level sets approach to segment the gray matter from MR images based on evolving two surfaces simultaneously. The main drawbacks of the deformable models and level sets include slow speed, strong sensitivity to initialization and incomplete segmentation in case of complicated object boundary with concavities. In order to overcome the limitations of the various prevalent methods, we propose a probabilistic graph cut algorithm and two simple yet effective geometric shape priors to segment the cerebral white matter. Graph cuts [9-11] have emerged as a popular medical image segmentation framework in the past decade and are also applied to problems like structure detections in the human brain [12]. The first contribution of the paper is the extension of the classical graph-cut algorithm [9] using a novel probabilistic framework. The probabilistic graph cut effectively handles low contrast between the cerebral white matter and the surrounding anatomical structures in the MRI data. Application of shape knowledge is often found very useful in the graph cut-based segmentation of medical images. For example, see the works of Freedman
and Zhang [13], Song et al. [14], and Zhang et al. [15]. Our second contribution is the incorporation of two simple yet effective geometric shape priors in the probabilistic graph cut algorithm. The two shape constraints are used for modeling the skull and the frontal lobe. In [13-15, 17], the shape constraints are added in the form of an additional energy term. In contrast, we employ the shape constraints to generate a subgraph (i.e. select a part of the image). The probabilistic graph cut algorithm is first run on the entire graph (i.e. the whole image) and then on the constraints-based subgraph. Given the complicated shape of the cerebral white matter, our focus in this paper is on fast and accurate segmentation in individual 2D slices (chosen in an unbiased manner) from input MRI sequences. Our proposed method is extremely fast, highly accurate and requires little user inputs.

2. METHODS

In this paper, we follow the framework of [9] with certain important modifications. An image is modeled as a weighted undirected graph \( G = G(V, E) \). Let \( P \) denote the set of all pixels. Each pixel \( p \in P \) constitutes a node/vertex in \( G \). In addition, two special terminal nodes, called the ‘source’ \( s \) and the ‘sink’ \( t \) are considered [16]. There are two types of edges/links in the present network, namely, the \( t \)-links (\( T \)) and the \( n \)-links (\( N \)). The \( t \)-links connect all the individual pixels \( p \) to the source node \( s \) and the sink node \( t \). A neighborhood \( Ne(p) \) is assumed for each pixel \( p \). In this paper, we have used the 8-neighborhood, i.e., \( Ne(p) = 8 \). The \( n \)-links are constructed between each pixel \( p \) and its neighboring pixels \( q \) i.e. \( q \in Ne(p) \). So, we can write:

\[
V = P \cup s \cup t
\]

\[
E = T \cup N
\]

2.1. Probabilistic Graph Cut

Let \( A \) define a segmentation, i.e., classification of all pixels into either “object” or “background”. So, the energy function can be written as [9]:

\[
E(A) = B(A) + \lambda R(A)
\]

(2)

where the boundary properties are given by the term \( B(A) \) and the region properties are given by the term \( R(A) \). Mathematically, they can be expressed as:

\[
B(A) = \sum_{p,q \in Ne(p)} B(p,q)
\]

(3)

\[
R(A) = \sum_{p \in P} R_p(A_p)
\]

(4)

Let \( O, B \) and \( I_p \) be the histogram of the object seeds, histogram of the background seeds and intensity of any pixel \( p \), respectively. Further, let \( Pr(I_p|O) \) and \( Pr(I_p|B) \) denote the probability of a certain pixel \( p \) to belong to the class “object” (i.e. white matter) and to the class “background” (i.e. other structures) respectively. We next define \( B(p,q) \), \( R_p(\text{“object”}) \) and \( R_p(\text{“background”}) \) in the following equations:

\[
B(p,q) = K_{(p,q)} \exp((I_p - I_q)^2 / 2\sigma^2) \times \frac{1}{d(p,q)}
\]

(5)

\[
R_p(\text{“object”}) = -K_p^o \ln(Pr(I_p|O))
\]

(6)

\[
R_p(\text{“background”}) = -K_p^b \ln(Pr(I_p|B))
\]

(7)

The term \( d(p,q) \) denotes distance between two pixels \( p \) and \( q \). Note that unlike [9], the three terms \( K_{(p,q)}, K_p^o, K_p^b \) appearing respectively in the equations (5), (6) and (7) are evaluated within the proposed probabilistic framework, as indicated below in the steps of the probabilistic graph cut algorithm.

Algorithm: Probabilistic Graph Cut

Step 1: Evaluation of \( K_{(p,q)} \) - Compare the probabilities of any two pixels \( p \in P \) and \( q \in Ne(p) \) to belong to the same segmentation class using the terms \( Pr(I_p|O), Pr(I_p|B), Pr(I_q|O) \) and \( Pr(I_q|B) \). If both \( p \) and \( q \) have higher probabilities to belong to the same class, assign \( K_{(p,q)} \) to \( K^o \). Otherwise, assign \( K_{(p,q)} \) to \( K^b \).

Step 2: Evaluation of \( K_p^o \) and \( K_p^b \) - Compare the probability of a single pixel \( p \) to belong to the segmentation class “object” or “background” using the terms \( Pr(I_p|O) \) and \( Pr(I_p|B) \). If \( p \) has a higher probability to belong to the “object” class, assign \( K_p \) to \( K_p^o \) and \( K_p \) to \( K_p^b \). Otherwise, assign \( K_p \) to \( K_p^b \) and \( K_p \) to \( K_p^o \).

Step 3: Apply probabilistic graph cut using the equations (2) to (7).

2.2. Geometric Shape Priors

Two simple yet effective geometric shape priors [17] are applied to extract a subgraph \( G_1 = G(V_1, E_1) \) by pruning the original graph \( G = G(V, E) \). Though more sophisticated methods for identifying the above subgraph exist, e.g. GVF snakes [18], application of such techniques would require more user intervention and a larger computation time. In sharp contrast, the current approach does not need any further user inputs (other than seeds) and has very little computational overhead. The skull is modeled as a semicircle with center \( (\alpha, \beta) \) and radius \( R \). The equation for the semi-circle in the present situation is given by:

\[
(x - \alpha)^2 + (y - \beta)^2 = R^2, \quad y \geq \beta
\]

(8)

The frontal lobe is modeled by an ellipse, with the same center as that of the hemisphere in equation (8), semi-major axis \( a \) and semi-minor axis \( b \). The equation for the ellipse is given by:

\[
\frac{(x - \alpha)^2}{a^2} + \frac{(y - \beta)^2}{b^2} = 1
\]

(9)

From the anatomical observation, the potential target pixels are always inside the skull and outside the frontal lobe. Let
(p_x, p_y) be the coordinates of any pixel p. We select only a portion of the pixels P, say P_1 (i.e. P_1 \subset P) as members of V_1, which satisfy the following two geometric constraints:

\[(p_x - \alpha)^2 + (p_y - \beta)^2 - R^2 < 0, p_y \geq \beta\]  \hspace{1cm} (10)

\[(x - \alpha)^2 + (y - \beta)^2 - 1 > 0\]  \hspace{1cm} (11)

Once the vertices are selected, the \(t\)-links (T_i) and the \(n\)-links (N_i) are constructed. It is obvious that T_i \subset T and N_i \subset N. So, we can write:

\[V_1 = P_1 \cup s \cup t\]  \hspace{1cm} (12a)

\[E_1 = T_i \cup N_i\]  \hspace{1cm} (12b)

The subgraph construction has two distinct advantages. Firstly, the time-complexity of the Ford-Fulkerson algorithm, used to find a minimum cut, is reduced considerably. The augmenting paths are detected using the Edmonds-Karp algorithm [19]. The time-complexity of the Edmonds-Karp method, on the original graph \(G = G(V,E)\) is \(O(VE^2)\) and that on the subgraph \(G_i = G_i(V_i,E_i)\) is \(O(V_iE_i^2)\). Let us assume that: \(V_j = \gamma V\) and \(E_j = \delta E\), where \(\gamma\) and \(\delta\) are two fractions. So, we can rewrite the time-complexity of the Edmonds-Karp algorithm on the subgraph \(G_i\) as \(O(\gamma\delta^2VE^2) < O(VE^2)\), the time-complexity of the same algorithm on the original graph \(G\). The second advantage, illustrated in the next section, is that the proposed construction has significantly improved the segmentation accuracy.

3. EXPERIMENTAL RESULTS

We have experimented with 20 different T1-weighted MRI datasets. A single 2D slice (of size 256 x 256 pixels) is chosen from the 3D set in each case in a completely unbiased manner. The parameter \(\lambda\) in equation (2) is experimentally chosen to be 70. The value of the parameter \(\sigma\) in equation (5), which can be treated as “camera noise,” is estimated as 3. The common center of the hemisphere and the ellipse, i.e. \((\alpha, \beta)\), which also happens to be the center-of-mass of the input image, is a part of the seeds inputted by the user. The other three shape parameters \(a, b\) and \(R\) (equations (8)-(9)) are extracted from the image using simple geometry. Finally, optimum values of the constants \(K_1\) and \(K_2\) are experimentally chosen as 10 and 1 respectively. We emphasize that the four parameters, namely, \(\lambda, \sigma, K_1\) and \(K_2\), once chosen are kept constant for all the 20 datasets. In figures 1 and 2, we show two sets of data with input (a), hand-segmented ground-truth (b), graph cut-based segmentation (c), probabilistic graph cut-based segmentation (d), and probabilistic graph cut with geometric priors-based segmentation (e) for each case. The figures clearly demonstrate that the results in 1(e) and 2(e) are much closer in appearance to the respective ground-truths 1(b) and 2(b). Dice coefficient is used for quantitative comparison of the performance of the different segmentation algorithms. For the dataset shown in figure 1(2), respective dice coefficients using graph cut, probabilistic graph cut and geometric shape priors added to the probabilistic graph cut are 62.23% (56.23%), 79.12% (71.65%) and 96.43% (93.52%). Table 1 shows that the value of the average dice coefficient is approximately 17% higher for the probabilistic graph cut algorithm compared to the one in [9]. Addition of the two geometric constraints further increases the value of the average dice coefficient by 18%. A similar behavior can be noticed from the same table for the minimum and maximum dice coefficients. The estimated dice coefficient is quite robust to the ratio of \(K_1\) and \(K_2\) above an optimal value of 10.

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<tr>
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<tbody>
<tr>
<td>Graph Cut [9]</td>
<td>55.92%</td>
<td>59.61%</td>
<td>64.42%</td>
</tr>
<tr>
<td>Prob. Graph Cut</td>
<td>70.84%</td>
<td>76.68%</td>
<td>81.12%</td>
</tr>
<tr>
<td>Prob. Graph Cut + Geom. Priors</td>
<td>93.17%</td>
<td>94.78%</td>
<td>96.69%</td>
</tr>
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Table 1. Accuracy of different segmentation algorithms

![Fig.1. Images for one dataset. (a) Input, (b) Ground-truth, (c) Graph Cut [9], (d) Probabilistic Graph Cut, (e) Probabilistic Graph Cut with Geometric Constraints.](image1)

![Fig.2. Images for a second dataset. (a) Input, (b) Ground-truth, (c) Graph Cut [9], (d) Probabilistic Graph Cut, (e) Probabilistic Graph Cut with Geometric Constraints.](image2)
The principal reason behind the failure of the graph cut algorithm [9] is due to poor contrast between the object (white matter) and to the background (other structures). This makes proper identification of inexpensive edges, a necessary criterion for the graph cut to work, very difficult. Probabilistic modification of the graph cut algorithm [9] and introduction of the shape constraints in the probabilistic graph cut framework significantly increase the chance of identifying the inexpensive edges and consequently make the whole segmentation process much more accurate. The average execution time for the probabilistic graph cut without the shape constraints is 5 sec. on an Intel(R) Core(TM)2 processor with a speed of 2.8 GHz. On the same machine, the average execution time drops down to only 1 sec. when the shape constraints are imposed.

4. CONCLUSION AND FUTURE SCOPE

In this paper, we address an important biomedical problem of segmentation of cerebral white matter from MRI data. A probabilistic modification of the capacity function used in [9], is proposed for that purpose. In addition to the probabilistic graph cut algorithm, two geometric shape constraints, one for the skull and the other for the frontal lobe, are employed. The proposed probabilistic graph cut algorithm is shown to improve the segmentation results, compared to [9]. Incorporation of the two geometric shape priors are found to i) further increase the accuracy of segmentation and ii) reduce the execution time. The framework, discussed in this paper, requires little user inputs in spite of neither using atlas nor applying any inhomogeneity correction in the MRI data

In future, we will extend this 2-D image segmentation work to 3-D. In addition to the standard graph cut [9], we plan to compare our results with some well-known non-graph based method like Expectation Maximization Segmentation Algorithm (EMS) algorithm [20]. We will also consider applying GVF snakes [18] for more accurate geometrical modeling of cerebral white matter region which can improve the accuracy of segmentation.

5. REFERENCES